

Problems of G and Multidimensional Models

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Abstract

The relations for \dot{G} in multidimensional model with Ricci-flat internal space and multicomponent perfect fluid are obtained. A two-component example: dust + 5-brane, is also considered.

1 Introduction

Dirac's Large Numbers Hypothesis (LNH) is the origin of many theoretical explorations of time-varying G . According to LNH, the value of \dot{G}/G should be approximately the Hubble rate. Although it has become clear in recent decades that the Hubble rate is too high to be compatible with experiment, the enduring legacy of Dirac's bold stroke is the acceptance by modern theories of non-zero values of \dot{G}/G as being potentially consistent with physical reality. There are three problems related to G , which origin lies mainly in unified models predictions: 1) absolute G measurements, 2) possible time variations of G , 3) possible range variations of G – non-Newtonian, or new interactions. For 1) and 3) see [4].

After the original *Dirac hypothesis* some new ones appeared and also some generalized *theories* of gravitation admitting the variations of an effective gravitational coupling. We may single out three stages in the development of this field:

1. Study of theories and hypotheses with variations of FPC, their predictions and confrontation with experiments (1937-1977).
2. Creation of theories admitting variations of an effective gravitational constant in a particular system of units, analyses of experimental and observational data within these theories [1] (1977-present).
3. Analyses of FPC variations within unified models [4] (present).

Different theoretical schemes lead to temporal variations of the effective gravitational constant:

1. Empirical models and theories of Dirac type, where G is replaced by $G(t)$.
2. Numerous scalar-tensor theories of Jordan-Brans-Dicke type where G depending on the scalar field $\sigma(t)$ appears.
3. Gravitational theories with a conformal scalar field arising in different approaches [1].
4. Multidimensional unified theories in which there are dilaton fields and effective scalar fields appearing in our 4-dimensional spacetime from additional dimensions [4]. They may help also in solving the problem of a variable cosmological constant from Planckian to present values.

A striking feature of most modern scalar-tensor and unification theories, is that they do not admit a unique and universal constant values of physical constants and of the Newtonian gravitational coupling constant G in particular. In this paper we briefly set out the results of some calculations which have been

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carried out for various theories, and we discuss various bounds that may be suggested by multidimensional theories. Although the bounds on \dot{G} and $\dot{G}(r)$ are in some classes of theories rather wide on purely theoretical grounds as a result of adjustable parameters, we note that observational data concerning other phenomena may place limits on the possible range of these adjustable parameters.

Here we limit ourselves to the problem of \dot{G} (for $\dot{G}(r)$ see [1, 2, 3, 4]). We show that the various theories predict the value of \dot{G}/G to be $10^{-12}/yr$ or less. The significance of this fact for experimental and observational determinations of the value of or upper bound on \dot{G} is the following: any determination with error bounds significantly below 10^{-12} will typically be compatible with only a small portion of existing theoretical models and will therefore cast serious doubt on the viability of all other models. In short, a tight bound on \dot{G} , in conjunction with other astrophysical observations, will be a very effective "theory killer."

Some estimations for \dot{G} were done long ago in the frames of general scalar tensor theories using values of cosmological parameters (Ω , H , q etc) [1, 4]. With modern values they predict \dot{G}/G at the level of $10^{-12}/yr$ and less (see recent estimations of A. Miyazaki [5], predicting time variations of G at the level of $10^{-13}yr^{-1}$) for the Machian-type cosmological solution in the Brans-Dicke theory).

The most reliable experimental bounds on \dot{G}/G (radar ranging of spacecraft dynamics [6]) and laser lunar ranging [10] give the limit of $10^{-12}/yr$.

2 \dot{G} in $(4+N)$ -dimensional cosmology with multicomponent anisotropic fluid

We consider here a $(4+N)$ -dimensional cosmology with an isotropic 3-space and an arbitrary Ricci-flat internal space. The Einstein equations provide a relation between \dot{G}/G and other cosmological parameters.

2.1 The model

Let us consider $(4+N)$ -dimensional theory described by the action

$$S_g = \frac{1}{2\kappa^2} \int d^{4+N}x \sqrt{-g} R, \quad (1)$$

where κ^2 is the fundamental gravitational constant. Then the gravitational field equations are

$$R_P^M = \kappa^2 (T_P^M - \delta_P^M \frac{T}{N+2}), \quad (2)$$

where T_P^M is a $(4+N)$ -dimensional energy-momentum tensor, $T = T_M^M$, and $M, P = 0, \dots, N+3$.

For the $(4+N)$ -dimensional manifold we assume the structure

$$M^{4+N} = R \times M_k^3 \times K^N, \quad (3)$$

where M_k^3 is a 3-dimensional space of constant curvature, $M_k^3 = S^3$, R^3 , L^3 for $k = +1, 0, -1$, respectively, and K^N is a N -dimensional compact Ricci-flat Riemann manifold.

The metric is taken in the form

$$g_{MN} dx^M dx^N = -dt^2 + a^2(t) g_{ij}^{(3)}(x^k) dx^i dx^j + b^2(t) g_{mn}^{(N)}(y^p) dy^m dy^n, \quad (4)$$

where $i, j, k = 1, 2, 3$; $m, n, p = 4, \dots, N+3$; $g_{ij}^{(3)}$, $g_{mn}^{(N)}$, $a(t)$ and $b(t)$ are, respectively, the metrics and scale factors for M_k^3 and K^N . For T_P^M we adopt the expression of the multicomponent (anisotropic) fluid form

$$(T_P^M) = \sum_{\alpha=1}^m \text{diag}(-\rho^\alpha(t), p_3^\alpha(t) \delta_j^i, p_N^\alpha(t) \delta_n^m). \quad (5)$$

Under these assumptions the Einstein equations take the form

$$\frac{3\ddot{a}}{a} + \frac{N\ddot{b}}{b} = \frac{\kappa^2}{N+2} \sum_{\alpha=1}^m [-(N+1)\rho^\alpha - 3p_3^\alpha - Np_N^\alpha], \quad (6)$$

$$\frac{2k}{a^2} + \frac{\ddot{a}}{a} + \frac{2\dot{a}^2}{a^2} + \frac{N\dot{a}\dot{b}}{ab} = \frac{\kappa^2}{N+2} \sum_{\alpha=1}^m [\rho^\alpha + (N-1)p_3^\alpha - Np_N^\alpha], \quad (7)$$

$$\frac{\ddot{b}}{b} + (N-1)\frac{\dot{b}^2}{b^2} + \frac{3\dot{a}\dot{b}}{ab} = \frac{\kappa^2}{N+2} \sum_{\alpha=1}^m [\rho^\alpha - 3p_3^\alpha + 2p_N^\alpha]. \quad (8)$$

The 4-dimensional density is

$$\rho^{\alpha,(4)}(t) = \int_K d^N y \sqrt{g^{(N)}} b^N(t) \rho^\alpha(t) = \rho^\alpha(t) b(t), \quad (9)$$

where we have normalized the factor $b(t)$ by putting

$$\int_K d^N y \sqrt{g^{(N)}} = 1. \quad (10)$$

On the other hand, to get the 4-dimensional gravity equations one should put $8\pi G(t)\rho^{\alpha(4)}(t) = \kappa^2\rho^\alpha(t)$. Consequently, the effective 4-dimensional gravitational “constant” $G(t)$ is defined by

$$8\pi G(t) = \kappa^2 b^{-N}(t) \quad (11)$$

whence its time variation is expressed as

$$\dot{G}/G = -N\dot{b}/b. \quad (12)$$

2.2 Cosmological parameters

Some inferences concerning the observational cosmological parameters can be extracted just from the equations without solving them [9]. Indeed, let us define the Hubble parameter H , the density parameters Ω^α and the “deceleration” parameter q referring to a fixed instant t_0 in the usual way

$$H = \dot{a}/a, \quad \Omega^\alpha = 8\pi G\rho^{\alpha,(4)}/3H^2 = \kappa^2\rho^\alpha/3H^2, \quad q = -a\ddot{a}/\dot{a}^2. \quad (13)$$

Besides, instead of G let us introduce the dimensionless parameter

$$g = \dot{G}/GH = -N\dot{b}/\dot{a}b. \quad (14)$$

Then, excluding b from (6) and (8), we get

$$\frac{N-1}{3N}g^2 - g + q - \sum_{\alpha=1}^m A^\alpha \Omega^\alpha = 0 \quad (15)$$

with

$$A^\alpha = \frac{1}{N+2} [2N+1+3(1-N)\nu_3^\alpha + 3N\nu_N^\alpha], \quad (16)$$

where

$$\nu_3^\alpha = p_3^\alpha/\rho^\alpha, \quad \nu_N^\alpha = p_N^\alpha/\rho^\alpha, \quad \rho^\alpha > 0. \quad (17)$$

When g is small we get from (15)

$$g \approx q - \sum_{\alpha=1}^m A^\alpha \Omega^\alpha. \quad (18)$$

Note that (18) for $N=6$, $m=1$, $\nu_3^1 = \nu_6^1 = 0$ (so that $A^1 = 13/8$) coincides with the corresponding relation of Wu and Wang [7] obtained for large times in case $k=-1$ (see also [8]).

If $k = 0$, then in addition to (18), one can obtain a separate relation between g and Ω^α , namely,

$$\frac{N-1}{6N}g^2 - g + 1 - \sum_{\alpha=1}^m \Omega^\alpha = 0 \quad (19)$$

(this follows from the Einstein equation $R_0^0 - \frac{1}{2}R = \kappa^2 T_0^0$, which is certainly a linear combination of (6)-(8).

The present observational upper bound on g is

$$|g| \lesssim 0.1 \quad (20)$$

if we take in accord with [6, 10]

$$|\dot{G}/G| \lesssim 0.6 \times 10^{-11} (y^{-1}) \quad (21)$$

and $H = (0.7 \pm 0.1) \times 10^{-11} (y^{-1}) \approx 70 \pm 10 (km/s.Mpc)$.

2.3 Two-component example: dust + $(N-1)$ -brane

Let us consider two component case: $m = 2$. Let the first component (called "matter") be a dust, i.e.

$$\nu_3^1 = \nu_N^1 = 0, \quad (22)$$

and the second one (called "quintessence") be a $(N-1)$ -brane, i.e.

$$\nu_3^2 = 1, \quad \nu_N^2 = -1. \quad (23)$$

We remind that as it was mentioned in [11] the multidimensional cosmological model on product manifold $\mathbf{R} \times M_1 \times \dots \times M_n$ with fields of forms (for review see [13]) may be described in terms of multicomponent "perfect" fluid [12] with the following equations of state for α -s component: $p_i^\alpha = -\rho^\alpha$ if p -brane worldvolume contains M_i and $p_i^\alpha = \rho^\alpha$ in opposite case. Thus, the field of form matter leads us either to Λ -term, or to stiff matter equations of state in internal spaces.

In this case we get from (18) for small g

$$g \approx q - \frac{2N+1}{N+2}\Omega^1 + 4\frac{N-1}{N+2}\Omega^2, \quad (24)$$

and for $k = 0$ and small g we obtain from (19)

$$1 - g \approx \Omega^1 + \Omega^2. \quad (25)$$

Now we illustrate the formulas by the following example when $N = 6$ (K^6 may be a Calabi-Yau manifold) and

$$-q = \Omega^1 = \Omega^2 = 0.5. \quad (26)$$

We get from (24)

$$g \approx -\frac{1}{16} \approx -0.06 \quad (27)$$

in agreement with (20).

In this case the second fluid component corresponds to magnetic (Euclidean) $NS5$ -brane (in $D = 10$ type I, Het or II A string models).

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